JOURNAL OF MATERIALS SCIENCE 26 (1991) 3575–3580

Crack tip opening and advance displacements of blunted cracks under plane stress

P. S. THEOCARIS, V. KYTOPOULOS
Department of Engineering Sciences, Athens National Technical University, P.O. Box 77230, Athens 175-10, Greece

The phenomenon of ductile blunting under plane stress conditions in cracked polycarbonate plates was studied. Because this phenomenon is intimately connected with the amount of crack opening displacement (CTOD) and its analogous phenomenon of crack tip advance displacement (CTAD), a study was undertaken of the mechanism of the development of blunting by evaluating the mode of evolution of CTOD and CTAD in the specimens. As study by scanning electron microscopy is limited to a thin layer of the surface of the specimen, this method is most convenient for studying blunting phenomena under plane-stress conditions. Interesting results were derived from these experiments and the characteristic properties of plane-stress blunting were determined.

1. Introduction

It is well established that before the initiation of propagation of a crack in a ductile material the initially sharp crack tip is more or less blunted depending not only on the properties of the material and the geometry of the specimen, but also on the mode of loading of the cracked plate. Although the mechanism of formation and development of blunting under conditions of plane strain has been either theoretically extensively studied [1–3] or experimentally examined [4], the same is not true for the state of plane stress.

Recent theoretical and experimental studies [5–8] on polycarbonate cracked plates have revealed some features of the formation of plane-stress blunting. Thus a theoretical model proposed previously [5] was confirmed by extensive experimental evidence with scanning electron microscopy [5–7], which has also shown some deviations from this model, especially for cases of mixed-mode loading of the cracked plates.

Because the phenomenon of blunting is intimately connected with the crack-tip opening displacement (CTOD), and also with the analogous notion of the crack-tip advance displacement (CTAD), quantities which constitute basic parameters for the mechanics of fracture, a more detailed study of the mechanism of formation of blunting may help simultaneously in the profound understanding of the notions of CTOD and CTAD where opinions are divided [9, 10].

The study of blunting in the SEM detects mainly the surface of the plate which, at least theoretically, is under conditions of plane stress. Such types of studies, therefore, are favoured in cases with thin specimens where their surfaces, together with the inside layers, tend to be under similar stress conditions and, therefore, the state of plane stress is favoured. Thus, measurements of the critical values of CTOD and CTAD are considered as being more direct than other types because they can avoid the various difficulties encountered in measurements of these quantities in thick specimens which are assumed to be under plane-strain conditions [11].

Thus, considering the experience gathered and the results derived from similar previous studies combined together with new observations and measurements *in situ* executed in the SEM with polycarbonate (PC) plates, this paper presents some clarifications of hitherto obscure facets of the phenomenon of blunting. The study is based on the qualitative examination of the morphological evolution of blunting, and also on a quantitative evaluation of the parameters expressing the phenomenon of blunting, that is the respective evolution of the CTOD and CTAD of an advancing ductile crack.

2. Experimental procedure

The specimens used in the tests were typical dog-bone tension specimens with length $l = 60$ mm, width $w = 7$ mm and thickness $t_1 = 3.0$ mm and $t_2 = 0.7$ mm. The specimens were prepared from thin polycarbonate of bisphenol-A plates, precracked using cuts made by razor blades. The specimens were polished and afterwards thoroughly cleaned and coated with a thin aluminium coating to prevent charging effects on their front faces by the impinging high-voltage electron beam. The scanning electron microscope was the Cambridge Scientific Instruments S4-10 model fitted with a stereoscan tensile specimen stage. SEM parameters were: beam voltage $V_B = 20$ kV, beam current 250 μA, filament current 2.5 A, jaw velocity 0.2 mm min^{-1}, whereas the maximum load capacity of the loading device was 2226 N. For the preparation of specimens, polycarbonate of bisphenol A (PCBA) plates were used. This material is a typical non-linear glassy polymer [12, 13], presenting a strong plastic deformation before fracture, but a strong elastic deformation before yielding. Although the similarity of the stress–strain curve between PCBA and low-carbon steels and other ductile metals is rather artificial [13], it is generally accepted that this material is convenient for modelling the plastic behaviour of ductile metals, by assuming that the non-linear region in the stress–strain curve of PCBA has the same effect on the stress distribution in the vicinity of the crack-tip, as the conception and development of plasticity in ductile and mildly strain-hardening metals.

The specimens were introduced inside the tensile stage of the microscope and submitted to a constant strain-rate loading applied in steps. In each step of the crack deformation, from the elastic range into the final step of introduction of blunting through development of some plastic deformation around the crack tips, the crack-tip zones were photographed using various magnification factors, varying between $\times$ 500 and $\times$ 2000.

For a better and easier study of the phenomena of elastic–plastic blunting, a series of steps of partial elastic unloading of the cracked plates was interposed between successive loading steps which were video-tape recorded by the special facility of TV scanning of the microscope, which were afterwards studied in detail. Because the scanning electron micrographs revealed interesting and novel phenomena, not as yet recorded, a series of relevant electron scanning micrographs are presented here.

3. Results and discussion

The various forms which the elastic–plastic blunting of ductile plates under plane strain conditions depend, in addition to the mode of stress state of the specimen, also on various accidental factors existing in the material of the plate in the vicinity of the crack tip which interfere, through the form of the artificial initial crack, with the mode of deformation of the crack. Thus, recent studies [6, 7] have shown that the forms which blunting can take under plane-strain or plane-stress conditions are liable to a statistical implication defining a dominating form of evolution of the phenomenon.

Figs 1 and 2 present the progressive formation of the dominant shape of the final blunting under plane-stress conditions in thin cracked plates of thickness $t_1 = 0.7$ mm and under mode I deformation. Two distinct forms of blunting appear. In Fig. 1 the flat-front blunting evoluted into one smooth re-entrant notch, whereas in Fig. 2 the double-kink blunting is evident. The three successive photographs in both figures show an early step of blunting (Figs 1a, 2a) which is reduced by unloading in Figs 1b and 2b, respectively, and then reloaded in Figs 1c and 2c. The development of a single or a double kink in either of these figures is self-explanatory.

Figs 3 and 4 show the evolution of blunting in cracked plates under plane-stress conditions and in a mixed-mode type of loading. The initial crack in Fig. 3 subtends an angle $\beta = 50°$ with the loading axis of the plate, whereas the crack-axis in Fig. 4 subtends an angle $\beta = 30°$. Fig. 4 is derived from the video-scanning recorder. Both figures show two early steps of loading and development of blunting with a double (Fig. 3) or single (Fig. 4) kink and Figs 3c and 4c correspond to the form of blunting after elastic unloading of the plates from the steps of Figs 3b and 4b, respectively.

It may be concluded from this evidence that the mode of development of ductile blunting under plane-stress conditions is similar for either mode I or mixed-mode loading of the cracked plates. On the contrary, the blunting mode for plates under plane-strain conditions is very different, as presented previously [6, 7] where specimens of thickness $t_2 = 3.0$ mm exhibited the so-called Japanese sword type of blunting under mixed-mode conditions of loading.

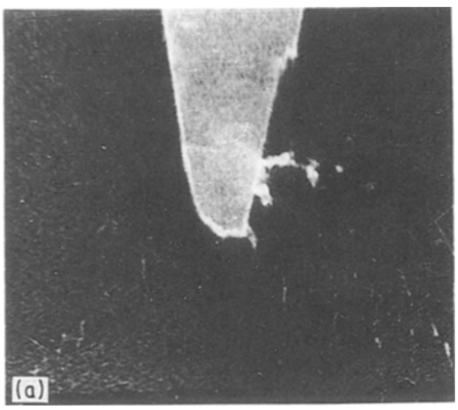
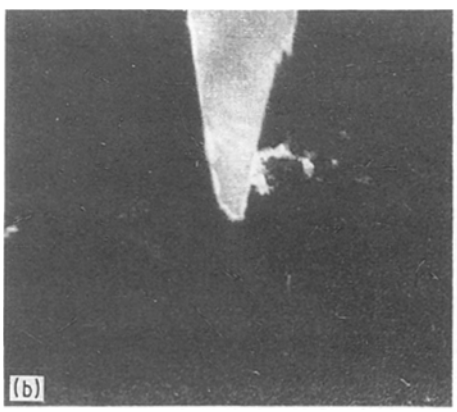
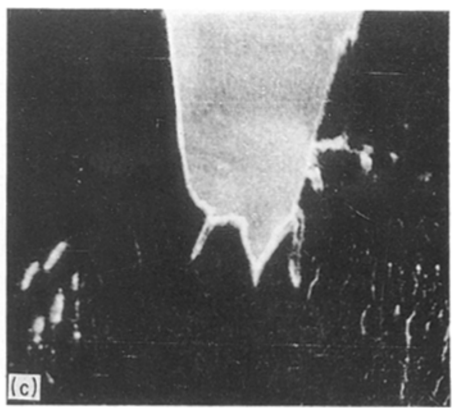
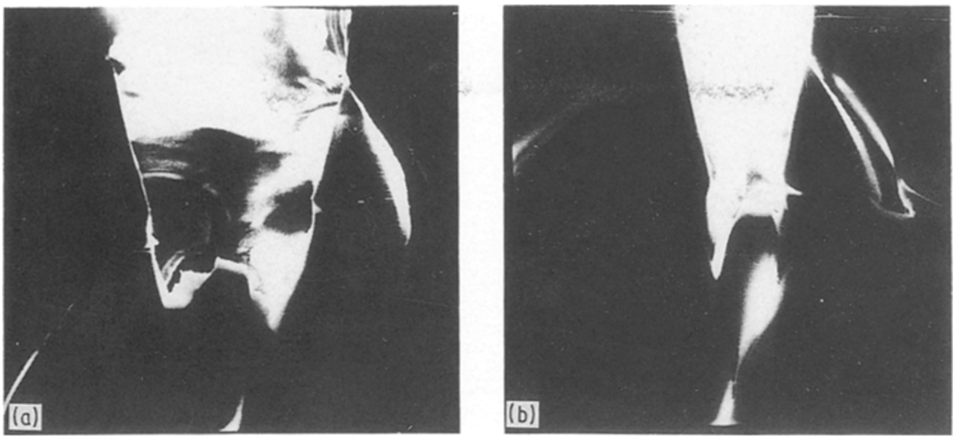


Figure 1 Evolution of the dominant "flat front" shape of the blunting under unloading–reloading conditions. (a, c) Loading, (b) unloading; $\beta = 90°$.







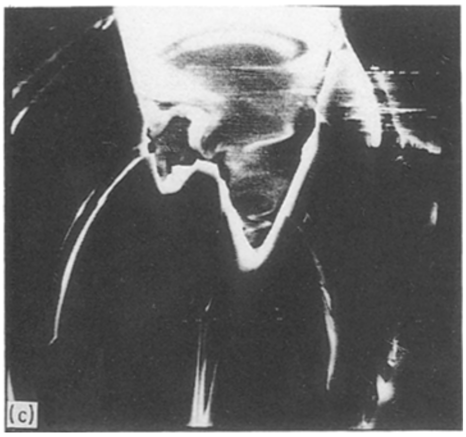


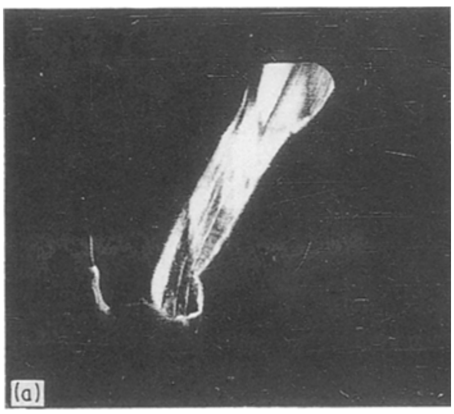
Figure 2 Evolution of the dominant "double-kink" shape of blunting under unloading–reloading conditions. (a) to (c) and β as Fig. 1.

For the measurement of CTOD and CTAD the same method was used as developed earlier [7]. According to this method a number of characteristic points surrounding the crack tip and corresponding to surface markings, point scratchings or spots, were used as reference points to define the initial position of the crack tip. It was shown [7] that the error in defining the position of the initial crack-tip by this method using SEM measurements was much smaller than the errors incurred using similar experimental methods.

Fig. 5 shows the results of variation of the CTOD with the respective values of CTAD for thin specimens under plane-stress conditions submitted to mode I or mixed-mode loading. For comparison, the same curve was plotted for plane-strain conditions of the plate and all these curves were compared with the ideal expression for this relationship given previously [6] and corresponding to a steady state increase of CTAD during the development of blunting from the initial crack tip. The CTOD $= f$(CTAD) relationship was found to be a linear one expressed by

$$\text{CTOD} \simeq \lambda(\text{CTAD}) \qquad (1)$$

The value of the multiplying factor, λ, for plane stress was found to be $\lambda = 5.0$, whereas for plane strain $\lambda = 3.18$, and the model in [6] corresponds to $\lambda = 2.0$. It is therefore clear from these values of the factor λ that even the thick specimens, which are assumed to be in a state approaching plane strain, present significant discrepancies for the value $\lambda = 2.0$ for the plane strain model, thus proving that the state of stress on



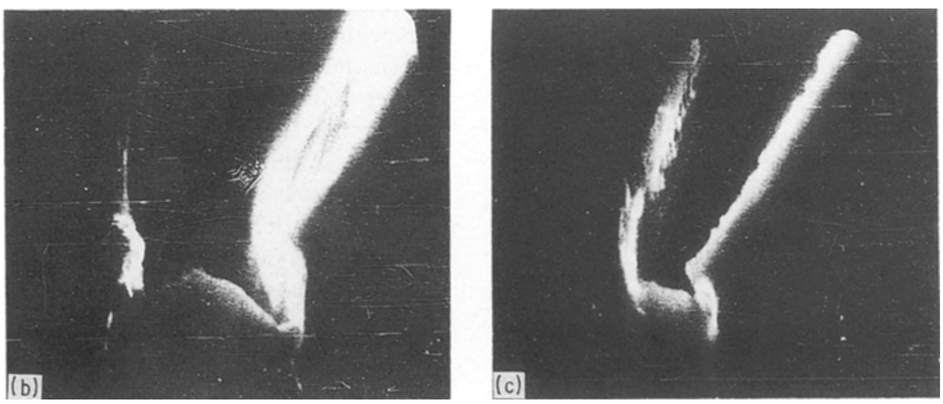
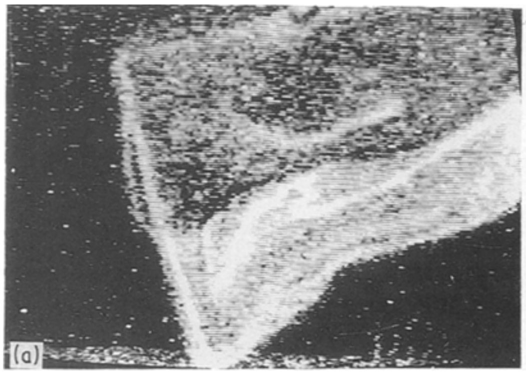
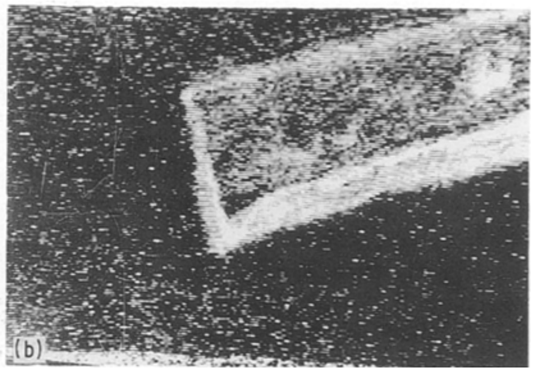


Figure 3 Evolution of a dominant shape of blunting in a mixed-mode type of unloading–reloading conditions (angle between initial crack and loading axis $\beta \simeq 50°$). (a, b) Loading, (c) unloading.





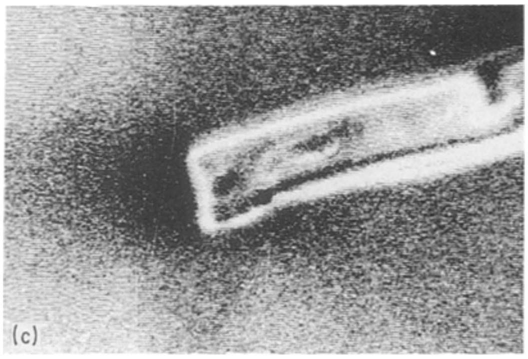


Figure 4 Evolution of a dominant shape of blunting in a mixed-mode type of unloading–reloading condition with $\beta \simeq 30°$. (a) to (c) as Fig. 3.

the surface of thick specimens is dominated by plane-stress conditions.

The CTAD in plane strain is mainly created by the contraction deformation field in front of the crack tip. Because for the ideal state of plane strain it is valid that the component of strain, ε_z, through the thickness of the specimen is equal to zero, it is also valid that $\varepsilon_x = -\varepsilon_y$ [2]. Thus if the plate is made from a rigid plastic material the region of the crack tip is under conditions of plastic incompressibility and therefore the crack will penetrate inside this region. This fact was shown not only theoretically, [14], but also experimentally by the creation of the so-called stretched zone [15, 16] for metallic plates.

However, with polycarbonate it was proved experimentally [7, 12, 13, 17] that its general behaviour

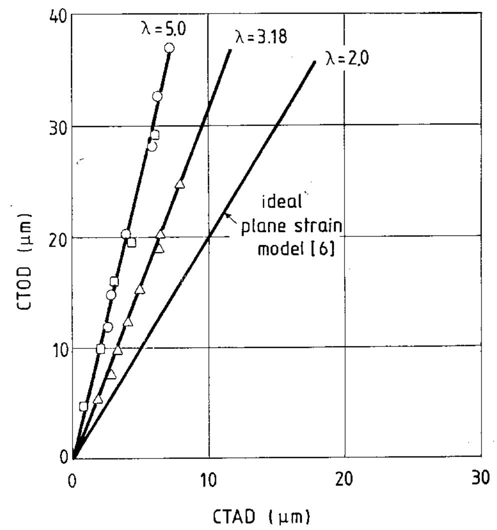


Figure 5 Variations of the CTOD with the respective values of CTAD for thin specimens. (○) Plane stress mode I, (□) plane stress mixed mode, (△) plane strain.

before yielding presents large elastic deformations in its stress–strain curve. But the local behaviour of polycarbonate around the crack tip presents analogous tendencies as again large experimental evidence has shown [18–20]. The same phenomenon clearly appears also in Fig. 6 where during the loading of the cracked plate together with the formation of blunting around the crack tip a dark-field also appears when the cracked specimen is examined in the SEM normal mode (Fig. 6a) and under Z-modulation arrangements (Fig. 6b). This dark field corresponds to a shallow dimple which disappears during unloading.

Thus, it is clear that a considerable elastic change in the thickness of the plate around the crack tip takes place, imposing a three-dimensional state of stress and strain. Thus, with the help of the so-called scanning electron elasticity the state of stress around the crack tip in PC plates is neither a clear plane stress nor a plane-strain condition during the first steps of loading of the plate. Therefore, due to the intense elastic contraction deformation field around the crack tip of PC plates there also exists an amount of elastic compressibility, i.e. there will be initially an increase in CTOD without an analogous increase in CTAD, which always requires a plastic incompressibility for its development.

However, from the successive loadings and unloadings executed during the tests with PC cracked plates, some of which are shown in Figs 1 and 2 for mode I and successive figures for mixed-mode conditions, it is clear that the elastic CTOD is sufficiently larger than its plastic part. Thus, the total CTOD for PC contains a large amount of elastic CTOD which is much larger than the plastic one. This is in contradiction with previous tests with PC where the elastic deformations measured by unloading were much smaller than the plastic ones [20].

The deviation between theory and experimental results concerning plane-strain tests may, in addition to the elastic compressibility of the material, also be due to the lateral faces' plane-stress phenomenon. Because observations by SEM are restricted to only the surface layers where $\varepsilon_z \neq 0$, the effective yield stress thus measured is much smaller than the yield stress in intermediate layers which are presumably under plane-strain conditions [21].

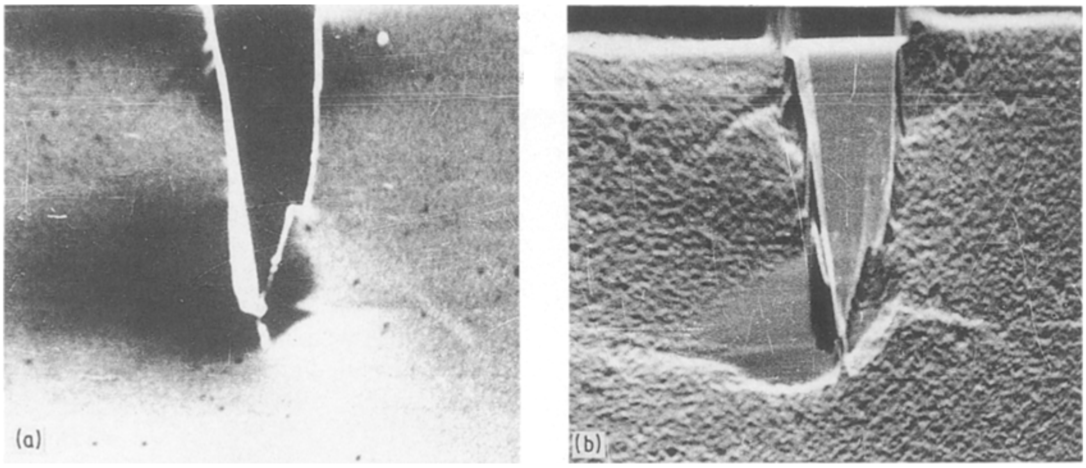


Figure 6 Visualization of change in the thickness around the crack tip by means of the so-called scanning electron elasticity: (a) normal mode, (b) Z-modulation (topographical mode). β = 90°.

However, since the specimens tested were very thin with a thickness of the order of $t = 0.7$ mm the two different states of stress present a tendency to coexist, thus creating an average plane-stress effect.

If in Equation 1, which is valid for ideal plane-strain conditions, we introduce the value $\lambda = \lambda_0$, whereas for ideal plane-stress $\lambda = \lambda_1$ and for the surface plane stress $\lambda = \lambda^*$ are true, then for the same CTOD it is valid that

$$\frac{\lambda_0}{\lambda^*} \simeq \frac{\lambda^*}{\lambda_1} \quad (2)$$

or

$$\lambda^* = (\lambda_0 \lambda_1)^{1/2} \quad (3)$$

Consequently, for the surface CTAD (S-CTAD) it is valid that

$$\text{S-CTAD} \simeq (\text{CTAD}_{\text{plane strain}} \times \text{CTAD}_{\text{plane stress}})^{1/2} \quad (4)$$

It is evident from Equations 3 and 4 that through the elastic crack-tip blunting compressibility we can derive general results concerning the mode of deformation of the plate and which one of the two extreme cases (total plane stress or total plane strain) is dominating the phenomenon of blunting and to what extent.

Indeed, Equation 4 indicates that the dominating state of stress in the cracked plate lies in between plane-stress and plane-strain conditions expressed by the geometric mean value between these two extreme states of stress. This fact is in agreement with the general remarks by McClintock [1] concerning the dominating state of stress on the surface of a material.

An important factor for the quantitative study of blunting is the correct evaluation of the critical CTOD where it is necessary to define accurately the point of initiation of propagation of the crack especially for thick specimens because of the already known tunnel effect [22]. This difficulty does not exist in our tests because we deal with thin specimens under plane-stress conditions. Thus in our blunted crack tips it is easy to define correctly the initiation of a through-the-thickness crack.

This is clearly indicated in Figs 1 and 2. Indeed, in Fig. 2 it is clear that a new crack is initiating at the right-hand side root of the corner, which becomes much larger than the left-hand side one, and the appearance of asymmetry between kinks is a safe criterion initiation of propagation of the crack. Thus, Fig. 2c yields the critical CTOD with high accuracy. As shown in Fig. 1, the situation is much simpler for the safe determination of the point of initiation of the crack propagation, because the profile of the blunted crack presents an almost smooth front. In this case the critical CTOD is normally larger in Fig. 2 compared with the respective CTOD in Fig. 1.

However, although both forms appear with almost the same statistical frequency in blunted cracks, the measurements of CTOD in this study always followed the procedure of Fig. 1. Fig. 1d shows an enlarged view of the state of blunting in Fig. 1c, where we can readily distinguish with the naked eye the interrelation between CTOD and CTAD from the flat front of the initial blunting remaining after the initiation of the propagation of the crack.

From these observations it may be concluded that in general the critical CTOD in plane stress is found to be smaller than those measured previously [6, 7]. The same is also valid for the critical CTOD under mixed-mode conditions.

From the series of photographs in Figs 3 and 4 it becomes obvious that the critical CTOD shows a tendency to diminish as the K_{II}/K_I ratio of the stress intensity factors increases. For this reason the measurements in Fig. 5 for mixed-mode conditions of loading of the cracks were extended up to an angle of 45°, after which the extent of blunting at the crack front was so diminished that the respective measurements of CTOD contained large errors. Thus, the measurements of cracks in Fig. 5 for mixed-mode conditions indicate that the λ-factor in Equation 1 and, therefore, the elastic crack-tip blunting compressibility, are independent.

Moreover, Fig. 4 indicates the mechanism of formation of blunting which follows a similar procedure as that indicated for mode I bluntings [6], even for mixed-mode conditions of loading [7] of the cracked plate. Indeed, for a certain angle of obliqueness of the crack and higher values of this angle, not only the form of blunting but also the mode of initiation of crack propagation follows a typical mixed-mode type [8]. This, however, ceases to be valid for angles of obliqueness smaller than this limit, and this is exemplified in Fig. 3.

4. Conclusions

Using the possibilities of theoretically infinite focusing on the crack tip by the scanning electron microscope and the simultaneous *in situ* detection of alternating loadings and unloadings in simple tension of thin specimens made of polycarbonate, qualitative observations and quantitative evaluations of the elastic–plastic behaviour in thin specimens under plane stress conditions, as well as the initiation and development of ductile blunting under conditions of either mode I or mixed-mode loading, have been made.

Qualitatively, the form of blunting in mode I deformation agrees with the theoretically established form based on the slip-lines method [5]. On the other hand, the form of blunting under mixed-mode conditions is not in complete agreement with the existing theory. For mixed mode conditions, especially, the form of blunting tends to follow the respective mixed-mode condition, a fact which is better discernible for smaller angles of obliqueness of the crack.

Based on a simple model introduced in this paper, the quantitative evaluation of the evolution of blunting could be undertaken based on accurate measurements of the respective CTOD and CTAD defined by this model. It has been proved that CTOD consists of an elastic and a plastic part, with the former one larger than the latter.

Furthermore, a new procedure of studying the state of stress valid around the crack tip was established. In this way it was shown that in between the two extreme cases of plane-stress and plane-strain conditions of the plate, there exists a broad spectrum of mixed states of stress and strain among which the state of the surface plane-stress conditions is lying. This state of stress may be expressed approximately by the geometric means of the extreme cases of plane stress and plane strain.

There was sufficient evidence gathered in this experimental study that a main cause of this variation in the state of loading of the plate is the large elastic compressibility of polycarbonate where especially in the close vicinity of the crack tip this quantity is expressed by the term elastic crack-tip blunting compressibility.

The increase of the ratio K_{II}/K_I of the stress intensity factors at the crack tip was found not to influence considerably the elastic crack-tip blunting compressibility, whereas an elastic–plastic mechanism of formation of blunting follows the respective mixed-mode condition of the plate. This increase in the K_{II}/K_I ratio, as a result, diminishes the elastic–plastic critical CTOD.

References

1. F. A. McCLINTOCK, in "Fracture: An Advanced Treatise", Vol. 3, edited by H. Liebowitz (Academic Press, New York, 1971) pp. 54, 175.
2. J. R. RICE and M. A. JOHNSON, in "Inelastic Behaviour of Solids", edited by M. F. Kanninen (McGraw Hill, New York, 1970) p. 641.
3. R. M. McMEEKING, *J. Mech. Phys. Solids* **25** (1977) 357.
4. S. P. RAWAL and J. GURLAND, *J. Met. Trans.* **8A** (1977) 691.
5. P. S. THEOCARIS, *Engng Fract. Mech.* **31** (1988) 255.
6. P. S. THEOCARIS, V. N. KYTOPOULOS and C. A. STASSINAKIS, *J. Mater. Sci.* **24** (1989) 1121.
7. P. S. THEOCARIS and V. N. KYTOPOULOS, *ibid.* **25** (1990) 997.
8. *Idem, Int. J. Fract. Mech.* **45** (1990) 329.
9. M. G. DAWES, ASTM-STP 668 (American Society for Testing and Materials, Philadelphia, Pennsylvania, 1979) p. 307.
10. W. D. CAO and X. P. LU, *Int. J. Fract.* **25** (1984) 33.
11. O. KODELNIK and H. P. STÜWE, *Engng Fract. Mech.* **21** (1985) 145.
12. P. S. THEOCARIS, *Int. J. Mech. Sci.* **17** (1975) 475.
13. *Idem, ibid.* **18** (1976) 171.
14. R. M. PELLOUX, *Engng Fract. Mech.* **1** (1970) 697.
15. W. A. SPITZIG, ASTM-STP 453 (American Society for Testing and Materials, Philadelphia, Pennsylvania, 1969).
16. K. H. SCHWALBE, *Engng Fract. Mech.* **6** (1974) 415.
17. P. S. THEOCARIS and E. E. GDOUTOS, *Exp. Mech.* **15**(5) (1975) 169.
18. P. S. THEOCARIS, in "Proceedings of IUTAM Symposium on Photoelasticity", Brussels, September 1973, edited by J. Kestens (Springer Verlag, Berlin, 1973) p. 146.
19. H. F. BRINSON, *Exp. Mech.* **10**(5) (1975) 72.
20. *Idem, ibid.* **11**(10) (1971) 467.
21. D. BROECK, "Elementary Engineering Fracture Mechanics" (Noordhoff International, Leyden, 1974) p. 265.
22. J. S. KE and H. W. LIU, *Engng Fract. Mech.* **5** (1973) 187.

Received 31 August 1989
and accepted 19 February 1990